

# Math Tricks All

I. Multiple of any two digit number with 11:-

ex.  $27 \times 11 = 2 \underline{9} 7$       ex.  $39 \times 11 = 4 \underline{2} 9$

$76 \times 11 = 8 \cancel{7} \underline{3} 6 = 836$

ex.  $52 \times 11 = 5 \underline{7} 2$

ex.  $98 \times 11 = 10 \underline{7} 8$

ex.  $87 \times 11 = 9 \underline{5} 7$

ex.  $97 \times 11 = 10 \underline{6} 7$

II. Square of any number ending with 5:-

ex.  $(45)^2 = \underline{20} \underline{25}$   
5 ↙ ↘ 5<sup>2</sup>

ex.  $(65)^2 = 4225$   
7 ↙ ↘ 5<sup>2</sup>

$$\begin{array}{r} 1 \\ 23 \\ \times 24 \\ \hline 552 \end{array}$$

ex.  $(95)^2 = 9025$   
10 ↙ ↘ 5<sup>2</sup>  
 $= \underline{24} \underline{35}^2$   
 $= \underline{24} \underline{552} \underline{25}$  ✓

ex.  $(215)^2 = 46225$   
22 ↙ ↘ 5<sup>2</sup>

ex.  $(115)^2 = 13225$   
12 ↙ ↘ 5<sup>2</sup>

ex.  $(795)^2 = 632025$   
 $(80-1)$   
80 ↙ ↘ 5<sup>2</sup>

### III. Multiplication of 2 two digit numbers :-

ex. 
$$\begin{array}{r} \overset{1}{\phantom{0}} \\ 23 \\ \times 42 \\ \hline 966 \end{array}$$

ex. 
$$\begin{array}{r} \overset{1}{\phantom{0}} \\ 33 \\ \times 22 \\ \hline 726 \end{array}$$

ex. 
$$\begin{array}{r} 25 \rightarrow 50 \\ \times 12 \\ \hline 300 \end{array} \quad \begin{array}{r} 50 \\ \times 6 \\ \hline 300 \end{array}$$

ex. 
$$\begin{array}{r} \overset{10}{\phantom{0}} \\ 96 \\ \times 39 \\ \hline 3744 \end{array}$$

$$\begin{array}{r} \cancel{11} \\ 76 \\ \times 89 \\ \hline 6764 \end{array}$$

$$\begin{array}{r} 63 \\ 53 \\ \hline \end{array}$$

ex. 
$$\begin{array}{r} \overset{13}{\phantom{0}} \\ 87 \\ \times 98 \\ \hline 8526 \end{array}$$

ex. 
$$\begin{array}{r} \overset{10}{\phantom{0}} \\ 79 \\ \times 84 \\ \hline 6636 \end{array}$$

### IV. Double - half method :-

ex. 
$$\begin{array}{r} 375 \times 16 = 6000 \\ \downarrow \quad \downarrow \\ 750 \times 8 \\ \downarrow \quad \downarrow \\ 1500 \times 4 \end{array}$$

$$\begin{array}{r} 650 \times 16 \\ \downarrow \quad \downarrow \\ = 1300 \times 8 \\ = 10400 \end{array}$$

ex. 
$$\begin{array}{r} 450 \times 12 = 5400 \\ \downarrow \quad \downarrow \\ 900 \quad 6 \end{array}$$

ex. 
$$\begin{array}{r} \checkmark \\ 235 \times 14 = 3290 \\ \downarrow \\ 470 \times 7 \end{array}$$

$$\text{ex. } \begin{array}{r} 350 \times 18 = 6300 \\ \downarrow \quad \downarrow \\ 700 \times 9 \end{array}$$

$$\text{ex. } \begin{array}{r} 334 \\ \underline{668} \times (5 \times \frac{2}{2}) \end{array}$$

$$\text{ex. } 210 \overline{840} \times \frac{(25 \times 4)}{4} = 21000$$

$$(35-65) = 3340$$

V. Square of any number near to 50 :-

$$\text{ex. } (48)^2 = \begin{array}{r} \uparrow 25-2 \\ \underline{2304} \\ \downarrow -2 \end{array}$$

$$\text{ex. } (39)^2 = \begin{array}{r} \textcircled{1} \\ \underline{1521} \\ \downarrow -11 \end{array} \quad \boxed{25}$$

$$\text{ex. } (63)^2 = \begin{array}{r} \textcircled{1} \\ \underline{3969} \\ \downarrow +13 \end{array} = \begin{array}{r} (38)^2 \\ \underline{1444} \\ \downarrow -12 \end{array}$$

$$\text{ex. } (59)^2 = \begin{array}{r} \downarrow +9 \\ \underline{3481} \end{array}$$

$$\text{ex. } (51)^2 = \begin{array}{r} \downarrow +1 \\ \underline{2601} \end{array}$$

$$\text{ex. } (34)^2 = \begin{array}{r} \textcircled{2} \\ \underline{1156} \\ \downarrow -16 \end{array}$$

$$\text{ex. } (64)^2 = \begin{array}{r} \textcircled{2} \\ \underline{4096} \\ \downarrow +14 \end{array}$$

## VI. Multiplication of any number with 15:-

ex.  $64 \times 15 = 960$

$$\begin{array}{r} 64 \\ + 32 \\ \hline 960 \end{array}$$

$$\begin{array}{r} 12.184 \\ 24.368 \times 15 = \\ \hline 365.52 \end{array}$$

ex.  $98 \times 15 = 1470$

$$\begin{array}{r} 98 \\ + 49 \\ \hline 1470 \end{array}$$

ex.  $0.008426 \times 15 = 0.12639$

$$\begin{array}{r} 0.008426 \\ 0.004213 \\ \hline 0.012639 \times 10 \end{array}$$

ex.  $32 \times 15 = 480$

$$\begin{array}{r} 32 \\ + 16 \\ \hline 480 \end{array}$$

ex.  $24684 \times 15 = 370260$

$$\begin{array}{r} 24684 \\ + 12342 \\ \hline 370260 \end{array}$$

ex.  $0.3468 \times 15 = 5.202$

$$\begin{array}{r} 0.3468 \\ + 0.1734 \\ \hline 0.5202 \times 10 \end{array}$$



# VII. Square of any two (or) three digit numbers :-

ex.  $(36)^2$

$$\begin{array}{r} 09 \quad 36 \\ + 36x \\ \hline 1296 \end{array}$$

$3 \times 6 \times 2$

ex.  $(76)^2$

$$\begin{array}{r} 49 \quad 36 \\ + 84x \\ \hline 5776 \end{array}$$

$(136)^2$

$$\begin{array}{r} 169 \quad 36 \\ + 156x \\ \hline 18496 \end{array}$$

$13 \times 4 \times 2$

$$\begin{array}{r} 169 \quad 16 \\ + 104x \\ \hline 17956 \end{array}$$

ex.  $(23)^2$

$$\begin{array}{r} 04 \quad 09 \\ + 12x \\ \hline 529 \end{array}$$

$(53)^2$

$$\begin{array}{r} 25 \quad 09 \\ + 30x \\ \hline 2809 \end{array}$$

$(213)^2$

$$\begin{array}{r} 441 \quad 09 \\ + 126x \\ \hline 45369 \end{array}$$

$(178)^2$

$$\begin{array}{r} 289 \quad 64 \\ + 272x \\ \hline 31684 \end{array}$$

ex.  $(123)^2$

$$\begin{array}{r} 144 \quad 09 \\ + 72x \\ \hline 15129 \end{array}$$

ex.  $(167)^2$

$$\begin{array}{r} 256 \quad 49 \\ + 224x \\ \hline 27889 \end{array}$$

VIII. Multiplication of any two numbers near to 100:- (80-120)

ex. 
$$\begin{array}{r} 103 \\ \times 105 \\ \hline 10815 \end{array}$$

 $\begin{array}{l} +3 \\ +5 \end{array}$

ex. 
$$\begin{array}{r} 93 \\ \times 96 \\ \hline 8928 \end{array}$$

 $\begin{array}{l} -7 \\ -4 \end{array}$

ex. 
$$\begin{array}{r} 104 \\ \times 87 \\ \hline 9048 \end{array}$$

 $\begin{array}{l} +4 \\ -13 \end{array}$ 

\* 
$$\begin{array}{r} 91(-52) \\ -100 \\ \hline 9048 \end{array}$$

ex. 
$$\begin{array}{r} 104 \\ \times 101 \\ \hline 10504 \end{array}$$

 $\begin{array}{l} +4 \\ +1 \end{array}$ 

\* 
$$\begin{array}{r} 114 +14 \\ \times 86 -14 \\ \hline 100(-196) \\ -2200 \\ \hline 9804 \end{array}$$

ex. 
$$\begin{array}{r} 88 \\ \times 91 \\ \hline 7908 \\ 80 \\ \hline = 8008 \end{array}$$

 $\begin{array}{l} -12 \\ -9 \end{array}$

ex. ✓ 
$$\begin{array}{r} 112 \\ \times 82 \\ \hline 9184 \end{array}$$

 $\begin{array}{l} +12 \\ -18 \end{array}$ 

\* 
$$\begin{array}{r} 94(-216) \\ -300 \\ \hline 9184 \end{array}$$
 ✓

# IX. Square root of a number which is a perfect square :-

ex.  $\sqrt{3364} = 58$

$\begin{array}{r} \boxed{5} \rightarrow 2 \\ \times 6 \rightarrow 8 \checkmark \\ \hline 30 \end{array}$

$\sqrt{7284} = 278$

$\begin{array}{r} \boxed{27} \rightarrow 2 \\ \times 28 \rightarrow 8 \uparrow \\ \hline 756 = 278 \end{array}$

ex.  $\sqrt{21609} = 147$

$\begin{array}{r} \boxed{14} \rightarrow 3 \\ \times 15 \rightarrow 7 \checkmark \\ \hline 210 \end{array}$

$29^2 \rightarrow 841$

ex.  $\sqrt{89401} = 299$

$\begin{array}{r} \boxed{29} \rightarrow 1 \\ \times 30 \rightarrow 9 \checkmark \\ \hline 870 \end{array}$

$$\text{ex. } \sqrt{15129} = \begin{array}{r} \boxed{12} \rightarrow 3 \checkmark \\ \times 13 \rightarrow 7 \\ \hline 156 \end{array} = 123$$

$$\begin{array}{r} 32 \\ \times 32 \\ \hline 1024 \end{array}$$

$$\text{ex. } \sqrt{3136} = \begin{array}{r} \boxed{5} \rightarrow 6 \checkmark \\ \times 6 \rightarrow 4 \\ \hline 30 \end{array} = 56$$

$$\begin{array}{r} 31 \\ \times 31 \\ \hline 961 \end{array}$$

$$\begin{array}{r} 33 \\ \times 33 \\ \hline 1089 \end{array}$$

$$\text{ex. } \sqrt{105625} = 325$$

**X. Square root of a number which is not a perfect square :- (approx)**

$$\text{ex. } \sqrt{27} = \sqrt{25 + 2} = 5 + \frac{2}{10} = 5.2$$

$$\boxed{\sqrt{a \pm b} = \sqrt{a} \pm \frac{b}{2\sqrt{a}}}$$

$$\begin{aligned} \text{ex. } \sqrt{60} &= \sqrt{64 - 4} = 8 - \frac{4}{16} = 8 - \frac{1}{4} \times \frac{25}{25} \\ &= 8 - 0.25 = 7.75 \end{aligned}$$

$$\begin{aligned} \text{ex. } \sqrt{340} &= \sqrt{324+16} = 18 + \frac{16}{36} = 18 + \frac{4}{9} \\ &= 18 + 0.44 = 18.44 \checkmark \end{aligned}$$

$$\text{ex. } \sqrt{29} = \sqrt{25+4} = 5 + \frac{4}{10} = 5.4$$

$$\begin{aligned} \text{ex. } \sqrt{55} &= \sqrt{49+6} = 7 + \frac{6}{14} = 7 + \frac{3}{7} \\ &= 7.42 \end{aligned}$$

$$\text{ex. } \sqrt{170} = \sqrt{169+1} = 13 + \frac{1}{26} = 13.04$$

$$\begin{aligned} \sqrt[3]{250} &= \sqrt[3]{144+6} = 12 + \frac{6}{24} = 12 + \frac{1}{4} \times \frac{25}{25} \\ &= 12.25 \text{ (approx.)} \end{aligned}$$

**VI. Cube root of a number which is a perfect cube :-**

$$\text{ex. } \sqrt[3]{\underline{300} \text{ } \underline{763}} = 67$$

$$\text{ex. } \sqrt[3]{\underline{704} \text{ } \underline{969}} = 89$$

$$\text{ex. } \sqrt[3]{\underline{3581} \text{ } \underline{577}} = 153$$

$$\text{ex. } \sqrt[3]{\underline{175} \text{ } \underline{616}} = 56$$

$$\text{ex. } \sqrt[3]{\underline{493} \text{ } \underline{039}} = 79$$

$$\text{ex. } \sqrt[3]{\underline{3176} \text{ } \underline{523}} = 147$$

$$\sqrt[3]{\underline{59} \text{ } \underline{319}} = 39$$

$1^3 \rightarrow 1$	$6^3 \rightarrow 216$
$2^3 \rightarrow 8$	$7^3 \rightarrow 343$
$3^3 \rightarrow 27$	$8^3 \rightarrow 512$
$4^3 \rightarrow 64$	$9^3 \rightarrow 729$
$5^3 \rightarrow 125$	$10^3 \rightarrow 1000$

$$11^3 \rightarrow 1331$$

$$12^3 \rightarrow 1728$$

$$13^3 \rightarrow 2197$$

$$14^3 \rightarrow 2744$$

$$15^3 \rightarrow 3375$$

XII. Dividing of two number's in which numerator is greater than the denominator and denominator is 99:-

$$\text{ex. } \frac{352}{99} = 3.55$$

$$\text{ex. } \frac{111}{99} = 1.12$$

$$\text{ex. } \frac{467}{99} = 4.71$$

$$\text{ex. } \frac{532}{99} = 5.37$$

$$\text{ex. } \frac{472}{99} = 4.76$$

$$\text{ex. } \frac{259}{99} = 2.61$$

$$\text{ex. } \frac{1234}{99} = 12.46$$

$$\text{ex. } \frac{2468}{99} = 24.92$$

XIII. Multiplication of any two numbers in which sum of their <sup>(units)</sup> ones place digits is 10 and remaining digits same:-

ex. 
$$\begin{array}{r} 84 \\ \times 86 \\ \hline 7224 \end{array}$$

ex. 
$$\begin{array}{r} 71 \\ \times 79 \\ \hline 5609 \end{array}$$

ex. 
$$\begin{array}{r} 113 \\ \times 117 \\ \hline 13221 \end{array}$$

ex. 
$$\begin{array}{r} 63 \\ \times 67 \\ \hline 4221 \end{array}$$

$$\begin{array}{r} 91 \\ \times 99 \\ \hline 9009 \end{array}$$

$$\begin{array}{r} 9009 \\ \hline 9009 \end{array}$$

ex. 
$$\begin{array}{r} 142 \\ \times 148 \\ \hline 21016 \end{array}$$

ex. 
$$\begin{array}{r} 96 \\ \times 94 \\ \hline 9024 \end{array}$$



XIV. Multiplication of any two numbers in which ones place digits are same and sum of tens place digits is 10 :-

ex. 
$$\begin{array}{r} 34 \\ \times 74 \\ \hline 2516 \end{array}$$

ex. 
$$\begin{array}{r} 82 \\ \times 22 \\ \hline 1804 \end{array}$$

ex. 
$$\begin{array}{r} 46 \\ \times 66 \\ \hline 3036 \end{array}$$

ex. 
$$\begin{array}{r} 63 \\ \times 43 \\ \hline 2709 \end{array}$$

ex. 
$$\begin{array}{r} 29 \\ \times 89 \\ \hline 2581 \end{array}$$

ex. 
$$\begin{array}{r} 91 \\ \times 11 \\ \hline 1001 \end{array}$$

ex. 
$$\begin{array}{r} 37 \\ \times 77 \\ \hline 2849 \end{array}$$

$$\begin{array}{r} 0 \quad 1 \quad 2 \quad 3 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \times 1 \quad 4 \\ \hline 1722 \end{array}$$

$$\begin{array}{r} 2 \quad 3 \quad 4 \quad 5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \times 1 \quad 7 \\ \hline 4165 \end{array}$$

XV. Multiplication of any number with a two digit number starting with 1 (Right neighbour rule):-

ex.

$$\begin{array}{r}
 \overset{1}{\curvearrowright} \overset{2}{\curvearrowright} \overset{3}{\curvearrowright} \overset{4}{\curvearrowright} \\
 0234 \\
 \times 12 \\
 \hline
 2808
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{2}{\curvearrowright} \overset{3}{\curvearrowright} \overset{1}{\curvearrowright} \\
 0685 \\
 \times 13 \\
 \hline
 8905
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{3}{\curvearrowright} \overset{2}{\curvearrowright} \overset{3}{\curvearrowright} \overset{4}{\curvearrowright} \overset{4}{\curvearrowright} \overset{2}{\curvearrowright} \\
 0846897 \\
 \times 14 \\
 \hline
 11856558
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{1}{\curvearrowright} \overset{2}{\curvearrowright} \overset{2}{\curvearrowright} \\
 0246 \\
 \times 14 \\
 \hline
 3444
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{2}{\curvearrowright} \overset{3}{\curvearrowright} \overset{4}{\curvearrowright} \overset{4}{\curvearrowright} \\
 03579 \\
 \times 15 \\
 \hline
 53685
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{5}{\curvearrowright} \overset{4}{\curvearrowright} \overset{4}{\curvearrowright} \overset{4}{\curvearrowright} \\
 09768 \\
 \times 15 \\
 \hline
 146520
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{1}{\curvearrowright} \overset{2}{\curvearrowright} \overset{2}{\curvearrowright} \overset{2}{\curvearrowright} \\
 6530 \\
 \times 31 \\
 \hline
 20243
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{1}{\curvearrowright} \overset{2}{\curvearrowright} \overset{2}{\curvearrowright} \overset{2}{\curvearrowright} \\
 4680 \\
 \times 21 \\
 \hline
 9828
 \end{array}$$

XVI . Multiplication of any number with a two digit number ending with 1 (Left neighbour rule):-

ex.

$$\begin{array}{r}
 \overset{1}{\leftarrow} \leftarrow \leftarrow \\
 16340 \\
 \leftarrow \uparrow \nearrow \\
 \times 21 \\
 \hline
 13314
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{4}{\leftarrow} \overset{5}{\leftarrow} \leftarrow \leftarrow \\
 24680 \\
 \leftarrow \uparrow \nearrow \\
 \times 61 \\
 \hline
 150548
 \end{array}$$

$$\begin{array}{r}
 \overset{2}{\leftarrow} \overset{2}{\leftarrow} \overset{2}{\leftarrow} \leftarrow \leftarrow \\
 234560 \\
 \leftarrow \uparrow \nearrow \\
 \times 41 \\
 \hline
 961696
 \end{array}$$

$$\begin{array}{r}
 \overset{5}{\leftarrow} \overset{4}{\leftarrow} \leftarrow \leftarrow \\
 19870 \\
 \leftarrow \uparrow \nearrow \\
 \times 51 \\
 \hline
 101337
 \end{array}$$

$$\begin{array}{r}
 \overset{3}{\leftarrow} \overset{3}{\leftarrow} \overset{2}{\leftarrow} \leftarrow \\
 87950 \\
 \times 31 \\
 \hline
 272645
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{1}{\leftarrow} \overset{2}{\leftarrow} \leftarrow \\
 1230 \\
 \leftarrow \uparrow \nearrow \\
 \times 21 \\
 \hline
 2583
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{5}{\leftarrow} \overset{2}{\leftarrow} \leftarrow \leftarrow \\
 457930 \\
 \times 51 \\
 \hline
 295443
 \end{array}$$

ex.

$$\begin{array}{r}
 \overset{6}{\leftarrow} \overset{4}{\leftarrow} \overset{3}{\leftarrow} \leftarrow \leftarrow \\
 6789650 \\
 \times 61 \\
 \hline
 4816865
 \end{array}$$

$$\frac{123}{139\overset{+1}{71}} = \frac{123}{140} = \frac{12.3}{14} = 0.\overset{\circ}{8}\overset{\circ}{8}\overset{\circ}{4}9^{\checkmark} \approx 0.8849$$

$$\frac{3.1}{7} = 0.\overset{\circ}{4}\overset{\circ}{4}\overset{\circ}{9}\overset{\circ}{2}7\text{---} = 0.449\overset{\checkmark}{3}$$

**XVII.** Division of any two numbers in which numerator is less than the denominator and the denominator ends with 9:-

ex.  $\frac{31}{69\overset{+1}{\rightarrow}} = \frac{31}{70} = \frac{3.1}{7} = 0.\overset{\circ}{4}\overset{\circ}{4}\overset{\circ}{9}\overset{\circ}{2}7\overset{\circ}{5}\overset{\circ}{3}\overset{\circ}{6}$

ex.  $\frac{67}{89\overset{+1}{\rightarrow}} = \frac{67}{90} = \frac{6.7}{9} = 0.\overset{\circ}{7}\overset{\circ}{5}\overset{\circ}{2}\overset{\circ}{8}\overset{\circ}{0}9$

ex.  $\frac{129}{139\overset{+1}{\rightarrow}} = \frac{129}{140} = \frac{12.9}{14} = 0.\overset{\circ}{9}\overset{\circ}{2}\overset{\circ}{8}\overset{\circ}{0}\overset{\circ}{5}\overset{\circ}{7}5$

ex.  $\frac{29}{39\overset{+1}{\rightarrow}} = \frac{29}{40} = \frac{2.9}{4} = 0.\overset{\circ}{7}\overset{\circ}{4}\overset{\circ}{3}\overset{\circ}{5}8 = 0.7436$

$$\text{ex. } \frac{76}{79 \xrightarrow{+1}} = \frac{76}{80} = \frac{7.6}{8} = 0.\overline{96202} = 0.9620$$

$$\text{ex. } \frac{111}{129 \xrightarrow{+1}} = \frac{111}{130} = \frac{11.1}{13} = 0.\overline{86046} = 0.8605$$

### XVIII. Multiplication of any number by 11 :-

$$\text{ex. } \underline{\underline{1234}} \times 11 = \underline{\underline{13574}}$$

$$\text{ex. } \underline{\underline{9876543}} \times 11 = 108645757$$

$$\text{ex. } 224466 \times 11 = 2469126$$

$$\text{ex. } 678769 \times 11 = 7466459$$

$$\text{ex. } 777888 \times 11 = 8556768$$

$$\text{ex. } 33557799 \times 11 = 369135789$$

$$\begin{array}{r} | \quad | \quad | \quad | \quad | \\ \underline{\underline{9876543}} \times 11 \\ = 107425857 \end{array}$$

$$87969565 \times 11$$

$$= 967665215$$

# ~~XX~~. Cube of any two digit number :-

~~XX~~

ex.

$$\begin{array}{r}
 \overbrace{(12)}^3 \\
 \swarrow \quad \searrow \\
 1^3 \textcircled{1} \quad 2 \quad 4 \quad \textcircled{8} 2^3 \\
 + 4 \quad 8 \\
 \hline
 1728
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|}
 \hline
 2 & 2 & 4 \\
 \hline
 1 & 2 & 3 \\
 \hline
 \end{array} \\
 \hline
 27552
 \end{array}$$

ex.

$$\begin{array}{r}
 \overbrace{(19)}^3 \\
 \swarrow \quad \searrow \\
 \frac{5}{2} \quad \frac{31}{9} \quad 8 \quad \textcircled{7} \textcircled{2} \textcircled{9} \\
 + 18 \quad 162 \\
 \hline
 6859
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|}
 \hline
 1 & 3 & 7 \\
 \hline
 2 & 1 & 5 \\
 \hline
 \end{array} \\
 \hline
 29455
 \end{array}$$

ex.

$$\begin{array}{r}
 \overbrace{(11)}^3 \\
 \swarrow \quad \searrow \\
 2 \quad 1 \quad 1 \quad 1 \\
 + 2 \quad 2 \\
 \hline
 1331
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|}
 \hline
 1 & 3 & 5 \\
 \hline
 2 & 4 & 6 \\
 \hline
 \end{array} \\
 \hline
 33210
 \end{array}$$

ex.

$$\begin{array}{r}
 \overbrace{(13)}^3 \\
 \swarrow \quad \searrow \\
 \frac{1}{1} \quad \frac{2}{3} \quad 9 \quad \textcircled{2} \textcircled{7} \\
 + 6 \quad 18 \\
 \hline
 2197
 \end{array}$$

ex.

$$\begin{array}{r}
 \overbrace{(15)}^3 \\
 \swarrow \quad \searrow \\
 \frac{2}{1} \quad \frac{8}{5} \quad 25 \quad \textcircled{12} \textcircled{5} \\
 + 10 \quad 50 \\
 \hline
 3375
 \end{array}$$

ex.

$$\begin{array}{r}
 \overbrace{(17)}^3 \\
 \swarrow \quad \searrow \\
 \frac{3}{1} \quad \frac{18}{7} \quad 49 \quad \textcircled{3} \textcircled{4} \textcircled{3} \\
 + 14 \quad 98 \\
 \hline
 4913
 \end{array}$$

## XX. Series addition of first n natural numbers:-

$$* 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\text{ex. } 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \frac{10 \times 11}{2} = 55$$

$\begin{array}{r} 5 \\ \times 11 \\ \hline 55 \end{array}$

$$\text{ex. } 1 + 2 + 3 + \dots + 200 = \frac{200 \times 201}{2} = 20100$$

$$\text{ex. } 1 + 2 + 3 + \dots + 25 = \frac{25 \times 26}{2} = 325$$

$$\text{ex. } 1 + 2 + 3 + \dots + 50 = \frac{50 \times 51}{2} = 1275$$

$$\text{ex. } 1 + 2 + 3 + \dots + 75 = \frac{75 \times 76}{2} = 75 \times 38 = 2850$$

$$\text{ex. } 1 + 2 + 3 + \dots + 49 = \frac{49 \times 50}{2} = 49 \times 25 = 1225$$

$$\text{ex. } 1 + 2 + 3 + \dots + 99 = \frac{99 \times 100}{2} = 99 \times 50 = 4950$$

$$6 + 6 = 12$$

# XVI. Series addition of Squares of first n natural numbers:-

$$* 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{ex. } 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{5 \times 6 \times 11}{6} = 55$$

$$\text{ex. } 1^2 + 2^2 + 3^2 + \dots + 200^2 = \frac{200 \times 201 \times 401}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + 50^2 = \frac{50 \times 51 \times 101}{6} = 2686700$$

$$\text{ex. } 1^2 + 2^2 + 3^2 + \dots + 25^2 = \frac{25 \times 26 \times 51}{6}$$

$$= 25 \times 221$$

$$= 5525$$

$$\text{ex. } 1^2 + 2^2 + 3^2 + 4^2 + \dots + 75^2 = \frac{75 \times 76 \times 151}{6}$$

$$= 950 \times 151$$

$$= 143450$$

$$\begin{array}{r} 25 \\ 17 \\ \hline 425 \\ \times 401 \\ \hline 42925 \\ \times 67 \\ \hline 2807 \\ 2406 \\ \hline 26867 \end{array}$$

$$\begin{array}{r} 221 \\ \times 25 \\ \hline \end{array}$$

$$\begin{array}{r} 1105 \\ 442 \\ \hline \end{array}$$

$$5525$$

$$151$$

$$\times 95$$

$$755$$

$$\begin{array}{r} 1359 \\ \hline 14345 \end{array}$$



$$\begin{aligned}
 \text{ex. } 1^2 + 2^2 + 3^2 + \dots + 49^2 &= \frac{49 \times \overset{25}{\cancel{50}} \times 99}{\cancel{2}}^{33} \\
 &= 1225 \times 33 = 1225 \times 3 \times 11 \\
 &= 3675 \times 11 = 40425
 \end{aligned}$$

$$\begin{aligned}
 \text{ex. } 1^2 + 2^2 + 3^2 + \dots + 99^2 &= \frac{\overset{33}{\cancel{99}} \times \overset{50}{\cancel{100}} \times 199}{\cancel{2}} \\
 &= 33 \times 50 \times 199 \\
 &= 3 \times 11 \times 9950 \\
 &= 11 \times 29850 \\
 &= 328350
 \end{aligned}$$

XVII. Series addition of cubes of first  $n$  natural numbers :-

$$* 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

$$\text{ex. } 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = \left( \frac{5 \times 6}{2} \right)^2 = 225$$

$$\text{ex. } 1^3 + 2^3 + 3^3 + \dots + 200^3 = \left( \frac{200 \times 201}{2} \right)^2$$

$$1^3 + 2^3 + 3^3 + \dots + 50^3 = \left( \frac{50 \times 51}{2} \right)^2 = (20100)^2$$

$$= \left( \frac{1275}{2} \right)^2 = 1625625 = 404010000$$

$$\text{ex. } 1^3 + 2^3 + 3^3 + \dots + 25^3 = \left( \frac{25 \times 26}{2} \right)^2 = (325)^2$$

$$= 105625$$

$$1^3 + 2^3 + 3^3 + \dots + 15^3 = \left( \frac{15 \times 16}{2} \right)^2 = (120)^2 = 14400$$

Handwritten calculations and multiplication:

$$\begin{array}{r} 25 \\ \times 51 \\ \hline 275 \\ 1275 \\ \hline 16256 \end{array}$$

2 25  
x 51  
-----  
1275  
2 25  
-----  
201  
x 201  
-----  
40401  
16256

32  
x 33  
-----  
1056

$$\begin{array}{r} 428 \\ \times 29 \\ \hline 812 \end{array}$$

$$\begin{aligned} \text{ex. } 1^3 + 2^3 + 3^3 + \dots + 75^3 &= \left(\frac{75 \times 76}{2}\right)^3 = (75 \times 38)^2 \\ &= (\underline{2850})^2 \\ &= 8122500 \end{aligned}$$

$$\begin{array}{r} 122 \\ \times 123 \\ \hline 15006 \end{array}$$

$$\begin{aligned} \text{ex. } 1^3 + 2^3 + 3^3 + \dots + 49^3 &= \left(\frac{49 \times 50}{2}\right)^2 = (49 \times 25)^2 \\ &= (\underline{1225})^2 \\ &= 1500625 \end{aligned}$$

$$\begin{aligned} \text{ex. } 1^3 + 2^3 + 3^3 + \dots + 99^3 &= \left(\frac{99 \times 100}{2}\right)^2 = (99 \times 50)^2 \\ &= (\underline{4950})^2 \\ &= 24502500 \end{aligned}$$

XXIII. Multiplication of a number with an equal number of 9:-

ex. 
$$\begin{array}{r} 654 \\ \times 999 \\ \hline \end{array}$$

$(653)346$   
 $654-1$        $\downarrow \downarrow \rightarrow 9-3$   
 $9-6 \quad 9-5$

ex. 
$$\begin{array}{r} 9994 \\ \times 9999 \\ \hline \end{array}$$

$(9993)0006$   
 $9994-1$        $\downarrow \downarrow \downarrow \rightarrow 9-3$   
 $9-9 \quad 9-9 \quad 9-9$

$(234)^{-1}$   
 $\times 999$   
 $\hline$   
 $233 \mid 766$   
 $= 233766$

ex. 
$$\begin{array}{r} 456789 \\ \times 9999999 \\ \hline \end{array}$$

$456788 \quad 543211$

ex. 
$$\begin{array}{r} 123 \\ \times 999 \\ \hline \end{array}$$

$(5697)^{-1}$   
 $\times 9999$   
 $\hline$

ex. 
$$\begin{array}{r} 65432 \\ \times 999999 \\ \hline \end{array}$$

$65431 \quad 34568$

ex. 
$$\begin{array}{r} 447 \\ \times 999 \\ \hline \end{array}$$

$5696, 4303$   
 $446553$

XXIV. Multiplication of a number with a higher number of 9's :-

$$\begin{array}{r} \text{ex. } \textcircled{045}^{-1} \\ \times 999 \\ \hline 044 \quad 955 \\ \hline = 44955 \end{array}$$

$$\begin{array}{r} \text{ex. } \textcircled{0067}^{-1} \\ \times 9999 \\ \hline 0066 \quad 9933 \\ \hline = 669933 \end{array}$$

$$\begin{array}{r} \textcircled{00765}^{-1} \\ \times 99999 \\ \hline 00764,99235 \\ \hline = 76499235 \end{array}$$

$$\begin{array}{r} \text{ex. } 0888 \\ \times 9999 \\ \hline 08879112 \\ \hline = 8879112 \end{array}$$

$$\begin{array}{r} \text{ex. } 0123 \\ \times 9999 \\ \hline 01229877 \\ \hline = 1229877 \end{array}$$

$$\begin{array}{r} \textcircled{09875}^{-1} \\ \times 99999 \\ \hline 09874,90125 \\ \hline = 987490125 \end{array}$$

$$\begin{array}{r} \text{ex. } 0888 \\ \times 9999 \\ \hline 08879112 \\ \hline = 8879112 \end{array}$$

$$\begin{array}{r} \text{ex. } 000363 \\ \times 999999 \\ \hline 000362999637 \\ \hline = 362999637 \end{array}$$

~~XXV~~. Multiplication of a number with a lower number of 9's :-

ex.  $654 \times 99 = 654 \times (100 - 1)$

$4986 \times 999 = 4986 \times (1000 - 1)$   
 $= 4986000$   
 $\quad - 4986$   
 $\hline 4981014$

$65400$   
 $\quad - 654$   
 $\hline 64746$

$467 \times 99$

$= 467 \times (100 - 1)$

ex.  $8754 \times 999 = 8754 \times (1000 - 1)$

$= 8754000$   
 $\quad - 8754$   
 $\hline 8745246$

$46700$   
 $\quad - 467$   
 $\hline 46233$

ex.  $123 \times 9 = 123 \times (10 - 1) =$

$1230$   
 $\times 123$   
 $\hline 1107$

ex.  $456 \times 99 = 45600$   
 $\quad - 456$   
 $\hline 45144$

$$\text{ex. } 6789 \times \overset{\vee}{9}\overset{\vee}{9}\overset{\vee}{9} = \begin{array}{r} 6789000 \\ - 6789 \\ \hline 6782211 \end{array}$$

$$\text{ex. } 80020 \times \overset{\vee}{9}\overset{\vee}{9}\overset{\vee}{9} = \begin{array}{r} 80020000 \\ - 80020 \\ \hline 79939980 \end{array}$$

$$\overset{\vee}{4}\overset{\vee}{1}\overset{\vee}{2}\overset{\vee}{3} \times \overset{\vee}{1}\overset{\vee}{1}\overset{\vee}{1} = \begin{array}{r} 457653 \\ \text{till here} \\ 1 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 1 \end{array}$$

XXVI. Multiplication of any number with 111 :-

$$\text{ex. } \underline{263} \times 111 = \underline{22533}$$

$$\text{ex. } \underline{20432} \times 111 = \underline{22358952}$$

$$\text{ex. } \underline{111} \times \underline{111} = \begin{array}{r} 12321 \end{array}$$

$$\text{ex. } \underline{2035} \times 111 = \underline{225885}$$

$$\text{ex. } \underline{1111} \times 1111 = \underline{123454321}$$

$$\text{ex. } \underline{1111} \times 1111 = \underline{1234321}$$

$$\underline{92756} \times 111 = \underline{10295916}$$

ex.  $90321 \times III = 10025631$



ex.  $6021203 \times III = 668353533$

$97531 \times III = 10825941$

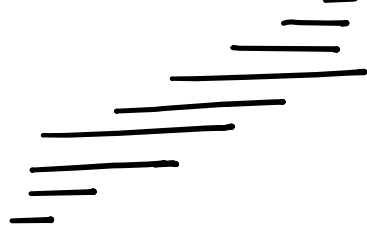
ex.  $24689 \times III$

$= 2740479$

$2740479$

XXVII. Multiplication of any number with III:-

ex.  $210432 \times IIII = 233789952$



ex.  $713524 \times IIII$

$= 792725164$

$792725164$

①

ex.  $12345 \times IIII = 13715295$



1 → 2 → 3 → 4 (till the end)

→ 3 → 2 → 1

12x

$57324 \times IIII = 63686964$



$$24683 \times IIII$$

$$= 27422813$$

ex.  $\underline{13579} \times IIII = 15086269$   $\times 2_1$

ex.  $\underline{234567} \times IIII = 260603937$   $\times 2_1$

ex.  $\underline{9876} \times IIII = 10972236$   $\times 2_3$

ex.  $\underline{887766} \times IIII = 986308026$   $\times 2_3$

$$123141 \times IIII = 136809651$$

$$\begin{array}{r} 115 \\ \times 15 \\ \hline 575 \\ 1125 \\ \hline 1725 \\ \times 5 \\ \hline 625 \\ \times XII \\ \hline = 6875 \end{array}$$

$$\begin{array}{r} 2055 \\ \times 875 \\ \hline 930 \end{array} \begin{array}{l} \swarrow \\ \nwarrow \end{array} \begin{array}{c} +55 \\ -125 \end{array}$$

$$\begin{array}{r} 985 \\ \times 1115 \\ \hline 1100 \end{array} \begin{array}{l} \swarrow -15 \\ \nwarrow +15 \end{array}$$

$$\begin{array}{r} 1100 \mid -1725 \\ -2 \mid 2000 \\ \hline \end{array}$$

$$\frac{-7}{923} \mid \frac{7000}{125} = 923 \mid 125 = 1098275$$

1098, 275

XXVIII. Multiplication of any two numbers near to 1000 (800 - 1200): -

ex.

$$\begin{array}{r} \overset{\vee}{950} \begin{array}{l} -50 \\ -20 \end{array} \\ \times 980 \\ \hline 930 \mid \textcircled{1000} \\ = 931000 \end{array}$$

ex.

$$\begin{array}{r} 1044 \begin{array}{l} +44 \\ -2 \end{array} \\ \times 998 \\ \hline 1042 \mid -88 \\ * \frac{-1}{1000} \\ \hline 1041912 \end{array}$$

ex.

$$\begin{array}{r} 879 \begin{array}{l} -121 \\ +4 \end{array} \\ \times 1004 \\ \hline 883 \mid -484 \\ * \frac{-1}{1000} \\ \hline 882516 \end{array}$$

ex.

$$\begin{array}{r} 977 \begin{array}{l} -23 \\ -10 \end{array} \\ \times 990 \\ \hline 967 \mid 230 \\ = 967230 \end{array}$$

ex.

$$\begin{array}{r} 93 \begin{array}{l} -7 \\ +5 \end{array} \\ \times 105 \\ \hline 98 \mid -35 \\ * \frac{-1}{100} \\ \hline 9765 \end{array}$$

$$\text{ex. } \begin{array}{r} 200 \\ \times 1020 \\ \hline \end{array} = 224000$$

$$1220 \mid 4000 = 12204000$$

$$\text{ex. } \begin{array}{r} 230 \\ \times 1003 \\ \hline \end{array} = 233690$$

$$1233 \mid 690 = 1233690$$

~~XXIX~~. Sum of Squares of two consecutive numbers :-

$$\begin{aligned} \text{ex. } \underline{50^2} + \underline{51^2} &= (\underline{50+51}) \times \textcircled{50} + \underline{51} \\ &= 101 \times 50 + 51 \\ &= 5050 + 51 = 5101 \end{aligned}$$

$$\begin{aligned} \text{ex. } 72^2 + 73^2 &= (72+73) \times 72 + 73 \\ &= \underline{145} \times 72 + 73 \\ &= 290 \times 36 + 73 \\ &= 10440 + 73 = 10513 \end{aligned}$$

$$\begin{array}{r} 29 \\ \times 36 \\ \hline 10440 \end{array} \quad \$4$$

$$\begin{aligned} \text{ex. } 30^2 + 31^2 &= (30+31) \times 30 + 31 \\ &= 61 \times 30 + 31 = 1830 + 31 = 1861 \end{aligned}$$

$$\begin{aligned} \text{ex. } 100^2 + 101^2 &= (100+101) \times 100 + 101 \\ &= 20100 + 101 = 20201 \end{aligned}$$

$$\begin{aligned} \text{ex. } 40^2 + 41^2 &= (40+41) \times 40 + 41 \\ (\checkmark) (40-1) \times (\checkmark) (70-1) &= (81 \times 40) + 41 = 3240 + 41 \\ &= 3281 \end{aligned}$$

$$\begin{aligned} \text{ex. } 79^2 + 80^2 &= (79+80) \times 79 + 80 \\ = 9800 - 40 - 70 + 1 &= 159 \times 79 + 80 \\ = 9800 - 210 + 1 &= (160-1) \times (80-1) + 80 \\ = 9801 - 210 = 9591 &= 12800 - 240 + 1 + 80 \\ &= 12560 + 81 = 12641 = 9591 \\ 69^2 + 70^2 &= (\checkmark) (69+70) \times (\checkmark) 69 + 70 = \textcircled{139} \times 69 + 70 \\ &= \begin{array}{r} 9591 \\ + 70 \\ \hline 9661 \end{array} \end{aligned}$$

$$179 \times 89 = (180 - 1) \times (90 - 1)$$

$$= 16200 - 270 + 1 = 16201 - 270$$

$$89^2 + 90^2 =$$

XXX. Special series addition :-  $(89 + 90) \times 89 + 90$

ex.      ①            ②            ③            ④

$$2 + 22 + 222 + 2222$$

$$= 2 \times (1234) = 2468$$

$$= 179 \times 89 + 90$$

$$= 15931 + 90$$

$$= 16021$$

ex.      ①            ②            ③            ④            ⑤

$$9 + 99 + 999 + 9999 + 99999$$

$$= 9 \times (12345) = 111105$$

ex.       $3 + 33 + 333 + 3333 = 3 \times 1234 = 3702$

ex.       $8 + 88 + 888 + 8888 = 8 \times 1234 = 9872$

ex.       $5 + 55 + 555 + 5555 = 5 \times 1234 = 6170$

ex.       $\checkmark 7 + \checkmark 77 + \checkmark 777 + \checkmark 7777 + \checkmark 77777 = 7 \times 12345 \checkmark$

$6 + 66 + 666 + 6666 = 6 \times 1234 = 7404$        $= 86415$